Pearson Edexcel Level 3 GCE

Friday 23 June 2023

Afternoon (Time: 1 hour 30 minutes) | Paper reference **9FM0/3D**

Further Mathematics

Advanced

PAPER 3D: Decision Mathematics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator,
Decision Mathematics Answer Book (enclosed)

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Write your answers for this paper in the Decision Mathematics answer book provided.
- **Fill in the boxes** at the top of the answer book with your name, centre number and candidate number.
- Do not return the question paper with the answer book.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the answer book provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

P72799A

P 7 2 7 9 9 A

Pearson

1.

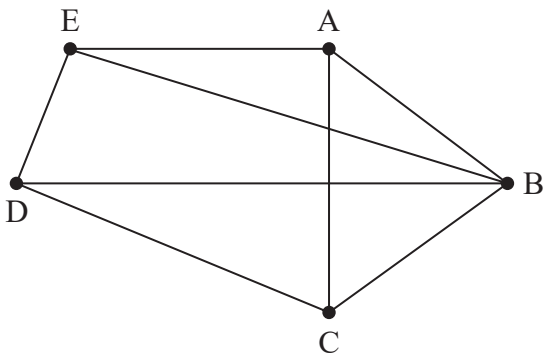

Figure 1

Figure 1 shows the graph G.

(a) State whether G is Eulerian, semi-Eulerian, or neither, giving a reason for your answer.

(1)

(b) Write down an example of a Hamiltonian cycle on G.

(1)

(c) State whether or not G is planar, justifying your answer.

(1)

(d) State the number of arcs that would need to be added to G to make the graph K_5

(1)

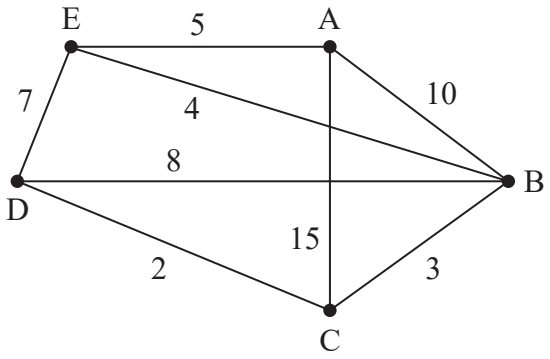

Figure 2

Direct roads between five villages, A, B, C, D and E, are represented in Figure 2. The weight on each arc is the time, in minutes, required to travel along the corresponding road. Floyd's algorithm is to be used to find the complete network of shortest times between the five villages.

(e) For the network represented in Figure 2, complete the initial time matrix in the answer book.

(1)

The time matrix after four iterations of Floyd's algorithm is shown in Table 1.

	A	**B**	**C**	**D**	**E**
A	–	10	13	15	5
B	10	–	3	5	4
C	13	3	–	2	7
D	15	5	2	–	7
E	5	4	7	7	–

Table 1

(f) Perform the final iteration of Floyd's algorithm that follows from Table 1, showing the time matrix for this iteration.

(2)

(Total for Question 1 is 7 marks)

2.

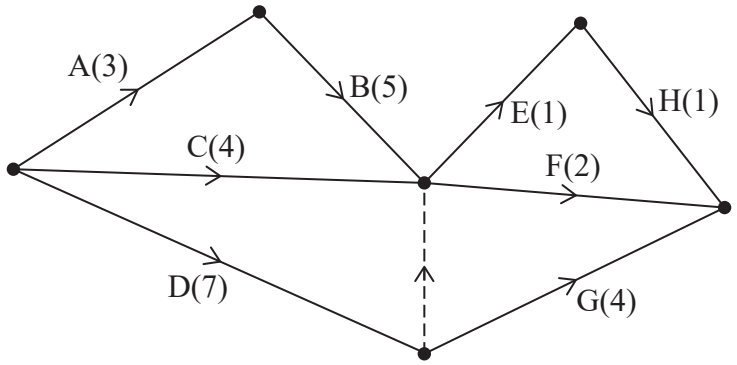

Figure 3

The network in Figure 3 shows the activities that need to be undertaken to complete a project. Each activity is represented by an arc and the duration, in hours, of the corresponding activity is shown in brackets.

(a) (i) Complete Diagram 1 in the answer book to show the early event times and the late event times.

(ii) State the minimum completion time of the project.

(3)

The table below lists the number of workers required for each activity in the project.

Activity	Number of workers
A	2
B	1
C	2
D	2
E	3
F	2
G	1
H	3

Each worker is able to do any of the activities. Once an activity is started it must be completed without interruption. It is given that each activity begins at its earliest possible start time.

(b) (i) On Grid 1 in the answer book, draw a resource histogram to show the number of workers required at each time.

(ii) Hence state the time interval(s) when six workers are required.

(4)

(Total for Question 2 is 7 marks)

3.

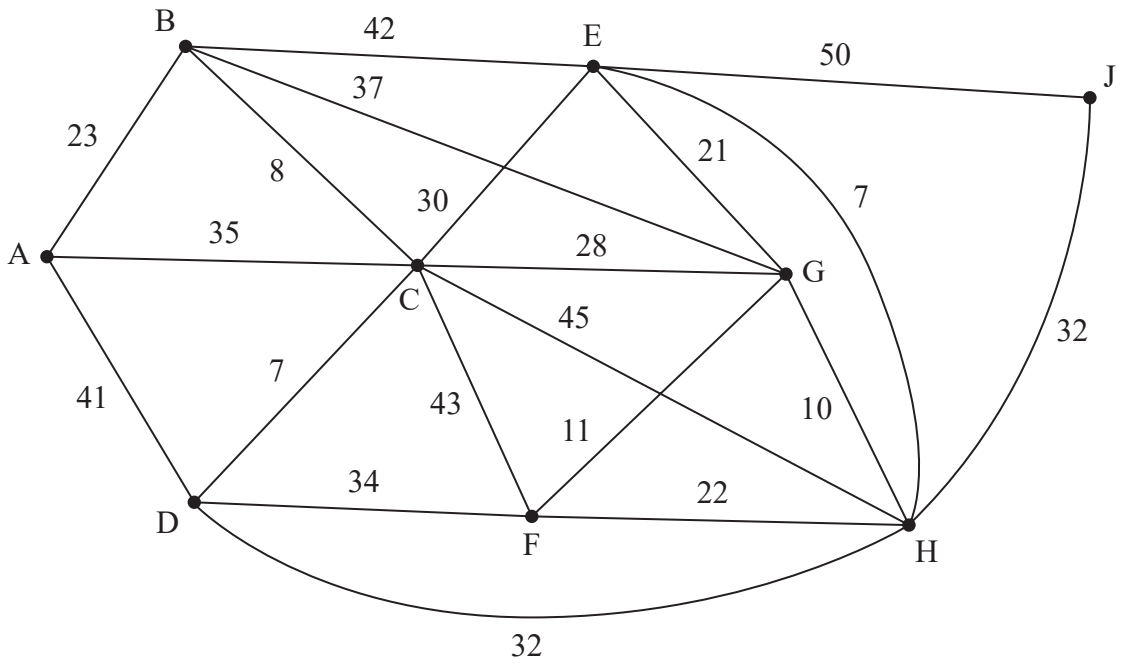

Figure 4

Figure 4 represents a network with nodes, A, B, C, D, E, F, G, H and J.
The number on each edge gives the length of the corresponding edge.

(a) (i) Use Dijkstra's algorithm to find the shortest path from A to J.

(ii) State the length of the shortest path from A to J.

(6)

One application of Dijkstra's algorithm has order n^2, where n is the number of nodes in the network.

It takes a computer 0.0312 seconds to find the shortest path from a given start node to a given end node in a network of 9 nodes.

(b) Calculate approximately how long it would take, in minutes, for the computer to find the shortest path from a given start node to a given end node for a network of 9000 nodes.

(2)

(Total for Question 3 is 8 marks)

4. The eleven distinct numbers listed below are to be packed into bins of size 40

15 22 3 9 23 x 5 4 18 20 13

It is known that x

- is an integer less than 40
- is the largest number in the list

(a) Explain why it **is not** possible to pack the numbers into 3 bins of size 40

(1)

Given that it **is** possible to pack the numbers into 4 bins of size 40

(b) determine the range of values for x

(2)

(c) Use the first-fit bin packing algorithm to determine how the numbers can be packed into bins of size 40

(3)

(d) Carry out a quick sort to produce a list of the numbers in descending order. You should show the result of each pass and identify your pivots clearly.

(4)

When the first-fit decreasing bin packing algorithm is applied to the list, neither the 15 nor the 13 is placed in the first bin.

(e) Determine the value of x. You must give reasons for your answer.

(2)

(Total for Question 4 is 12 marks)

BLANK PAGE

5.

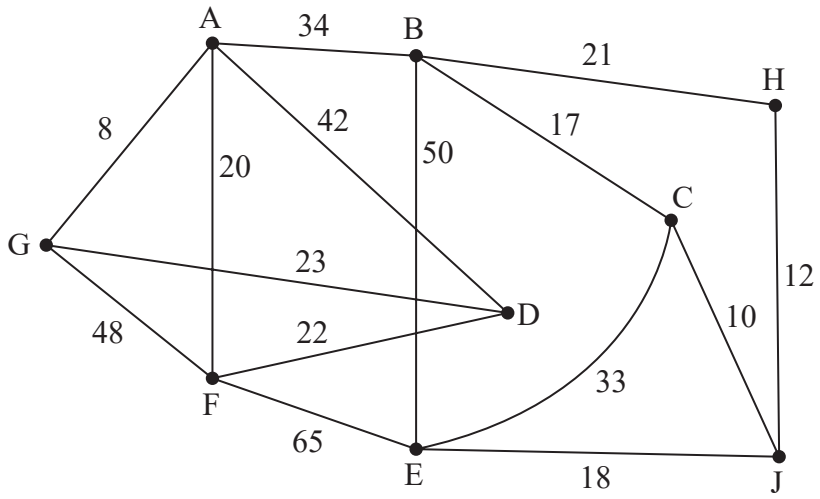

Figure 5

[*The total weight of the network is* 423]

Direct roads between nine towns, A, B, C, D, E, F, G, H and J, are represented in Figure 5. The number on each arc represents the length, in miles, of the corresponding road.

The table below shows the shortest distances, in miles, between the nine towns.

	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**J**
A	–	34	51	31	79	20	8	55	61
B	34	–	17	65	45	54	42	21	27
C	51	17	–	82	28	71	59	22	10
D	31	65	82	–	87	22	23	86	92
E	79	45	28	87	–	65	87	30	18
F	20	54	71	22	65	–	28	75	81
G	8	42	59	23	87	28	–	63	69
H	55	21	22	86	30	75	63	–	12
J	61	27	10	92	18	81	69	12	–

Table of shortest distances

A route is needed that minimises the total distance required to traverse each road at least once.

The route must start at F and finish at J.

(a) (i) By considering the pairings of all relevant nodes, find the roads that would need to be traversed twice.

(ii) State the total length of this route.

(5)

(b) Starting at A, use Prim's algorithm to find the minimum spanning tree for the table of shortest distances. You must state the order in which you select the arcs of your tree.

(3)

Pete needs to visit all nine towns, starting and finishing in the same town, and wishes to minimise the total distance he travels.

(c) Starting at G, use the nearest neighbour algorithm on the table of shortest distances to find an upper bound for the length of Pete's route. Write down the route that gives this upper bound.

(2)

(d) By deleting G and all of its arcs, find a lower bound for the length of Pete's route.

(2)

Pete decides to take the route he found in (c).

(e) Interpret the route in terms of the actual towns visited.

(1)

(Total for Question 5 is 13 marks)

6. The precedence table below shows the twelve activities required to complete a project.

Activity	Immediately preceding activities
A	–
B	–
C	–
D	A
E	A, B
F	D, E
G	A, B, C
H	F, G
I	D, E
J	D, E
K	F, G, I, J
L	I

(a) Draw the activity network described in the precedence table, using activity on arc. Your activity network must contain the minimum number of dummies only.

(5)

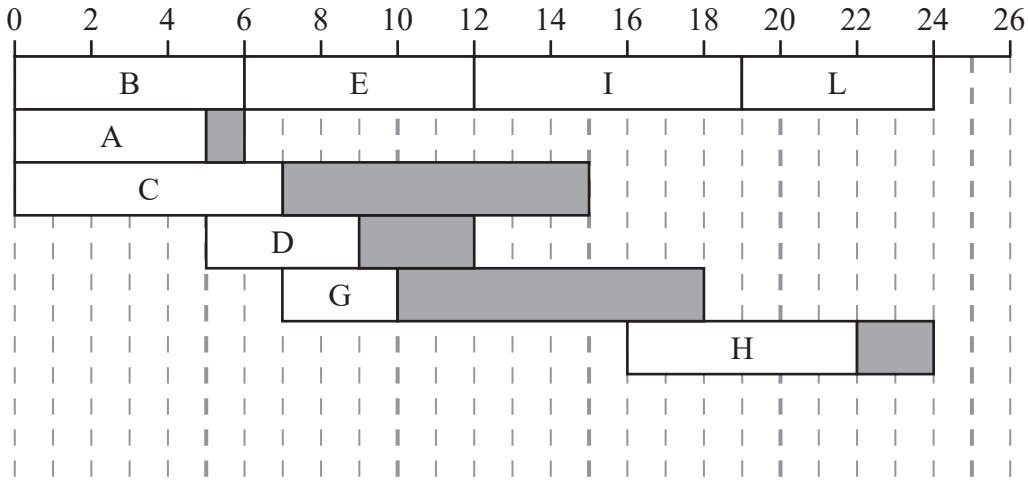

Figure 6

Figure 6 shows a partially completed cascade chart for the project. The non-critical activities F, J and K are **not** shown in Figure 6.

The time taken to complete each activity is given in hours and the project is to be completed in the minimum possible time.

(b) State the critical activities.

(1)

Given that the total float of activity F is 2 hours,

(c) state the duration of activity F.

(1)

The duration of activity J is x hours, and the duration of activity K is y hours, where $x > 0$ and $y > 0$

(d) (i) State, in terms of y, the maximum possible total float for activity K.

(ii) State, in terms of x and y, the total float for activity J.

(2)

(Total for Question 6 is 9 marks)

7. A publisher plans to produce three versions of the same book: a paperback, a hardcover, and a deluxe edition.

- Each paperback takes 4 minutes to print and 1 minute to bind
- Each hardcover takes 8 minutes to print and 5 minutes to bind
- Each deluxe edition takes 15 minutes to print and 12 minutes to bind

The printing machine is available for at most 150 hours and the binding machine must be used for at least 60 hours.

The publisher decides to produce

- at least 1600 books in total
- at least three times as many paperbacks as hardcovers

The profit on each paperback sold is £8, the profit on each hardcover sold is £20 and the profit on each deluxe edition sold is £40

Let x, y and z represent the number of paperbacks, hardcovers and deluxe editions produced.

(a) Formulate this as a linear programming problem, stating the objective and listing the constraints as simplified inequalities with integer coefficients.

(5)

The publisher decides to solve this linear programming problem by using the two-stage simplex method.

(b) Set up an initial tableau for solving this problem using the two-stage simplex method.

As part of your solution, you must show how

- the constraints have been made into equations by using slack variables, exactly two surplus variables and exactly two artificial variables
- the rows for the two objective functions are formed

(6)

The following tableau is obtained after two iterations of the first stage of the two-stage simplex method.

b.v.	x	y	z	s_1	s_2	s_3	s_4	a_1	a_2	Value
s_1	0	0	0	1	1	3	0	–1	–3	600
z	0	$\frac{4}{11}$	1	0	$-\frac{1}{11}$	$\frac{1}{11}$	0	$\frac{1}{11}$	$-\frac{1}{11}$	$\frac{2000}{11}$
x	1	$\frac{7}{11}$	0	0	$\frac{1}{11}$	$-\frac{12}{11}$	0	$-\frac{1}{11}$	$\frac{12}{11}$	$\frac{15600}{11}$
s_4	0	$\frac{40}{11}$	0	0	$\frac{1}{11}$	$-\frac{12}{11}$	1	$-\frac{1}{11}$	$\frac{12}{11}$	$\frac{15600}{11}$
P	0	$-\frac{4}{11}$	0	0	$-\frac{32}{11}$	$-\frac{56}{11}$	0	$\frac{32}{11}$	$\frac{56}{11}$	$\frac{204800}{11}$
I	0	0	0	0	0	0	0	1	1	0

(c) Taking the most negative number in the profit row to indicate the pivot column, perform one complete iteration of the second stage of the two-stage simplex method to obtain a new tableau. Make your method clear by stating the row operations you use.

(5)

After three iterations of the second stage of the two-stage simplex method, the following tableau is obtained.

b.v.	x	y	z	s_1	s_2	s_3	s_4	Value
s_2	0	0	0	1	1	3	0	600
z	0	0	1	$\frac{1}{10}$	0	$\frac{1}{2}$	$-\frac{1}{10}$	100
x	1	0	0	$-\frac{3}{40}$	0	$-\frac{9}{8}$	$-\frac{7}{40}$	1125
y	0	1	0	$-\frac{1}{40}$	0	$-\frac{3}{8}$	$\frac{11}{40}$	375
P	0	0	0	$\frac{29}{10}$	0	$\frac{7}{2}$	$\frac{1}{10}$	20500

Given that the publisher produces the optimal number of each version of the book,

(d) (i) state the maximum profit the publisher can earn,

(ii) find the number of hours the binding machine will be used.

(2)

(e) Give a reason why the publisher may not earn the profit stated in (d)(i).

(1)

(Total for Question 7 is 19 marks)

TOTAL FOR PAPER IS 75 MARKS

BLANK PAGE

BLANK PAGE